The measurement postulates of QM are redundant

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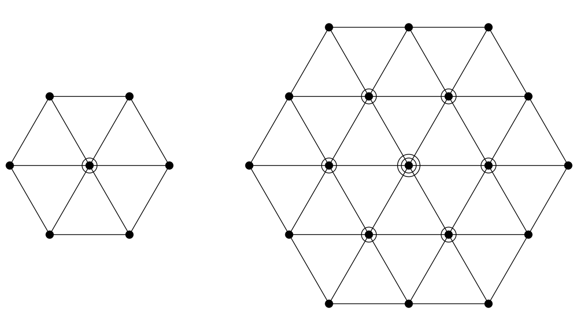


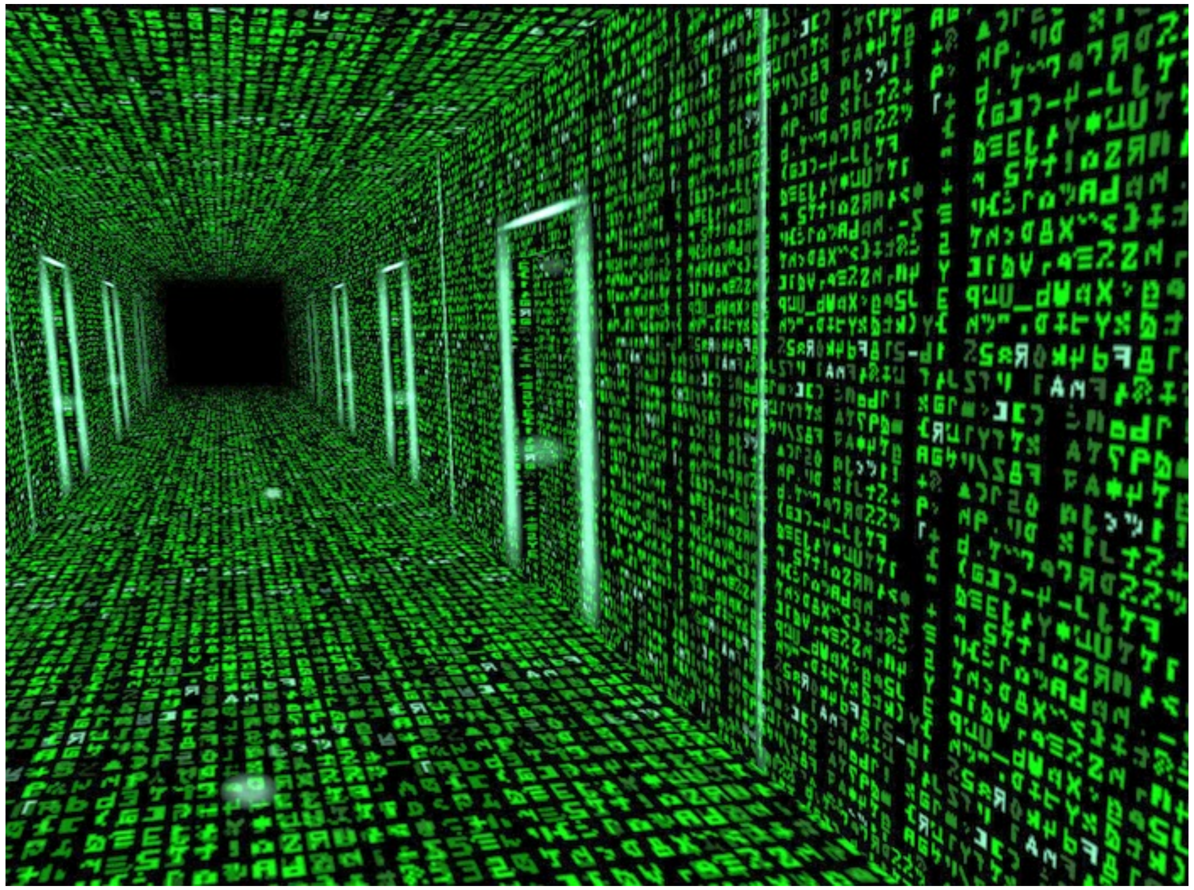
Quantum picture of reality

- We live in a superposition of worlds with copies of ourselves enjoying different lives.

Quantum picture of reality

- Atoms, electrons and photons are not actual reality, but states of knowledge.

- But then, what is reality?



Plan

- Next we look at the logical inner-structure of QM.
- Unveil the fact that the measurement postulates are a logical consequence of the rest of quantum postulates.
- That is, the measurement postulates are redundant.

Postulates of QT

States

$$\psi \in \mathrm{P}\mathbb{C}^d$$

Dynamics

$$\psi \mapsto U\psi, \quad U \in \mathrm{SU}(d)$$

Composite states

$$\mathbb{C}^d = \mathbb{C}^a \otimes \mathbb{C}^b$$

Measurements

$$P(Q|\psi) = \langle\psi|Q|\psi\rangle, \quad 0 \leq Q \leq \mathbb{1}$$

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Measurements

$$P(\mathbf{f}|\psi) = \mathbf{f}(\psi) \qquad\qquad \mathbf{f} \in \mathcal{F}_d$$

Example 1

$$P(\varphi_i|\psi) = \frac{\left|\langle\varphi_i|\psi\rangle\right|^4}{\sum_j \left|\langle\varphi_j|\psi\rangle\right|^4}$$

Example 2

$$\mathbf{f}_i(\varphi) = \mathrm{tr}\Big(F_i |\varphi\rangle\langle\varphi|^{\otimes 2}\Big) \qquad \sum_{i=1}^{n} F_i = P_+^a$$

Properties of $\mathcal{F}_d$

$$\mathbf{f} : \mathrm{P}\mathbb{C}^d \to [0,1]$$

$$\mathbf{f}^x \in \mathcal{F}_d \quad \Rightarrow \quad \sum_x p_x \, \mathbf{f}^x \in \mathcal{F}_d$$

$\mathrm{SU}(d)$ action $\quad U : \mathcal{F}_d \to \mathcal{F}_d$

$$U : \mathbf{f}(\psi) \mapsto \mathbf{f}(U\psi)$$

Alternative measurement postulate
→ alternative *SU(d)* representation

Properties of $\mathcal{F}_d$

$$\star : \mathcal{F}_a \times \mathcal{F}_b \to \mathcal{F}_{ab}$$

$$(\mathbf{f} \star \mathbf{g})(\psi \otimes \phi) = \mathbf{f}(\psi)\mathbf{g}(\phi)$$

$$\left(\sum_x p_x \, \mathbf{f}^x\right) \star \mathbf{g} = \sum_x p_x \left(\mathbf{f}^x \star \mathbf{g}\right)$$

$$(\mathbf{f} \circ U) \star \mathbf{g} = (\mathbf{f} \star \mathbf{g}) \circ (U \otimes \mathbb{1})$$

$$(\mathbf{f} \star \mathbf{g}) \star \mathbf{h} = \mathbf{f} \star (\mathbf{g} \star \mathbf{h})$$

Properties of $\mathcal{F}_d$

Assumption (possibility of state estimation). *Each finite-dimensional system* $\mathbb{C}^d$ *has a finite list of outcomes* $\mathbf{f}^1, \ldots, \mathbf{f}^k \in \mathcal{F}_d$ *such that knowing their value on any ensemble* (ψ_r, p_r) *allows to calculate any other OPF* $\mathbf{g} \in \mathcal{F}_d$ *on the ensemble* (ψ_r, p_r).

Theorem (measurement). *The only family of OPF sets $\mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4, \ldots$ and $\mathcal{F}_\infty$ equipped with a $\star$-product satisfying the "possibility of state estimation" assumption and conditions (7-14), has OPFs and $\star$-product of the form*

$$\mathbf{f}(\varphi) = \langle\varphi|F|\varphi\rangle \ , \tag{15}$$

$$(\mathbf{f} \star \mathbf{g})(\psi) = \langle\psi|F \otimes G|\psi\rangle \ , \tag{16}$$

for all $\varphi \in \mathbb{C}^a$ and $\psi \in \mathbb{C}^a \otimes \mathbb{C}^b$, where the $\mathbb{C}^a$-operator F satisfies $0 \le F \le \mathbb{1}$, and analogously for G.

Postulate (post-measurement state-update rule). *Each outcome is represented by a completely-positive linear map Λ related to the operator Q via*

$$\mathrm{tr}\Lambda(|\psi\rangle\langle\psi|) = \langle\psi|Q|\psi\rangle \ , \tag{2}$$

for all ψ. The post-measurement state after outcome Λ is

$$\rho = \frac{\Lambda(|\psi\rangle\langle\psi|)}{\mathrm{tr}\Lambda(|\psi\rangle\langle\psi|)} \ . \tag{3}$$

A (full) measurement is represented by the maps corresponding to its outcomes $\Lambda_1, \ldots, \Lambda_n$ whose sum $\sum_{i=1}^{n} \Lambda_i$ is a trance-preserving map.

Conclusions

- The quantum measurement postulate is the only possibility that is compatible with the dynamical part of QM.
- Hence, the content of the measurement postulate does not need to be postulated.

Conclusions

- This is a repeated pattern in the history of physics.
- Example 1: the rebranding of the symmetrization postulate as the spin-statistics theorem.
- Example 2: the derivation of the laws of thermodynamics form the principles of mechanics.

Thank you