Macroscopic Superpositions & The Quantum Nature of Gravity through Levitated Objects

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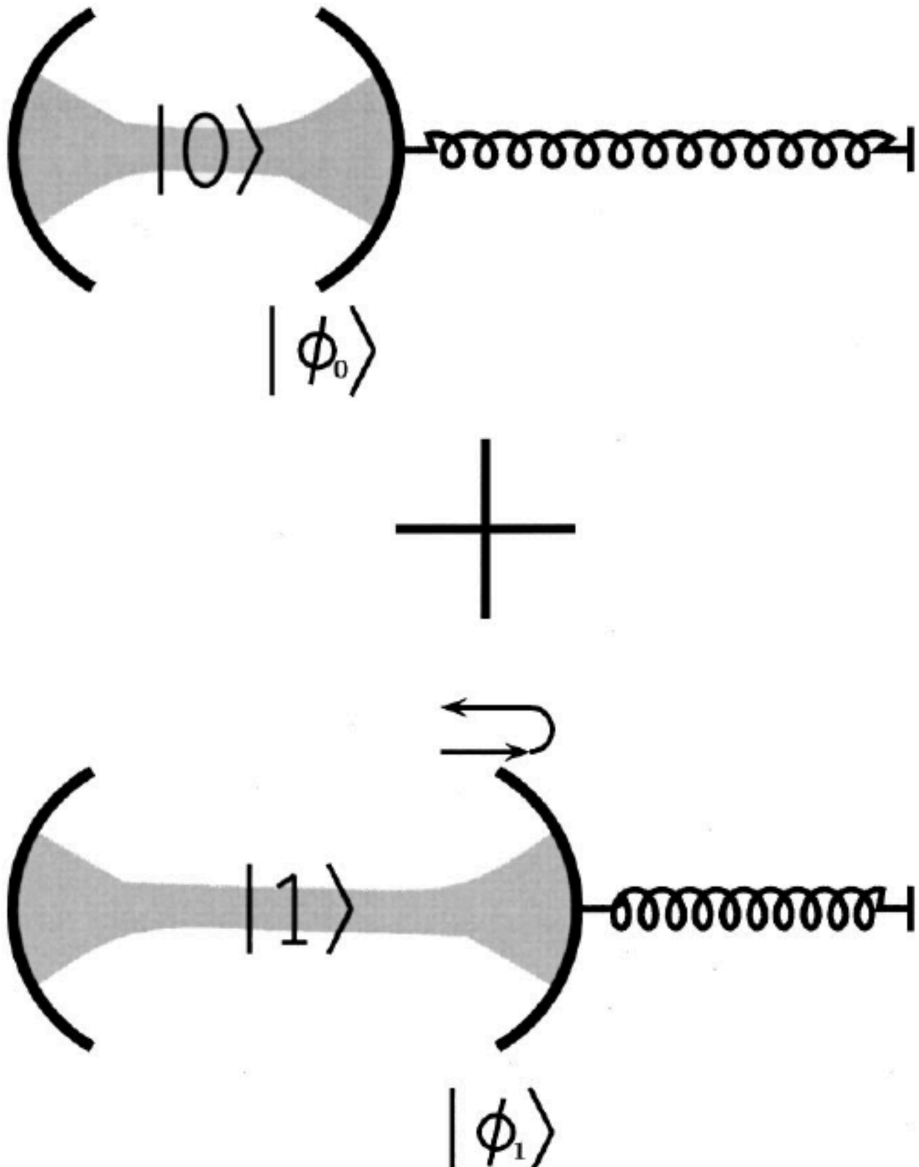
Based on:

- M. Scala, M. S. Kim, G. W. Morley, P. F. Barker, S. Bose, Phys. Rev. Lett. **111**, 180403 (2013).
- C. Wan, M. Scala, G. W. Morley, ATM. A. Rahman, H. Ulbricht, J. Bateman, P. F. Baker, S. Bose, M. S. Kim, Phys. Rev. Lett. 117, 143003 (2016).
- S. Bose, A. Mazumdar, G. W.Morley, H. Ulbricht, M. Paternostro, P. F. Barker, A. Geraci, M. S. Kim, G. J. Milburn, arXiv (soon)

***Tiny* Superpositions of a Macroscopic Object (the older idea was to investigate through the decoherence induced into the ancilla by the macrosopic object)**



S. Bose, K. Jacobs, P. L. Knight,
Phys. Rev. A 59 (5), 3204 (1999).

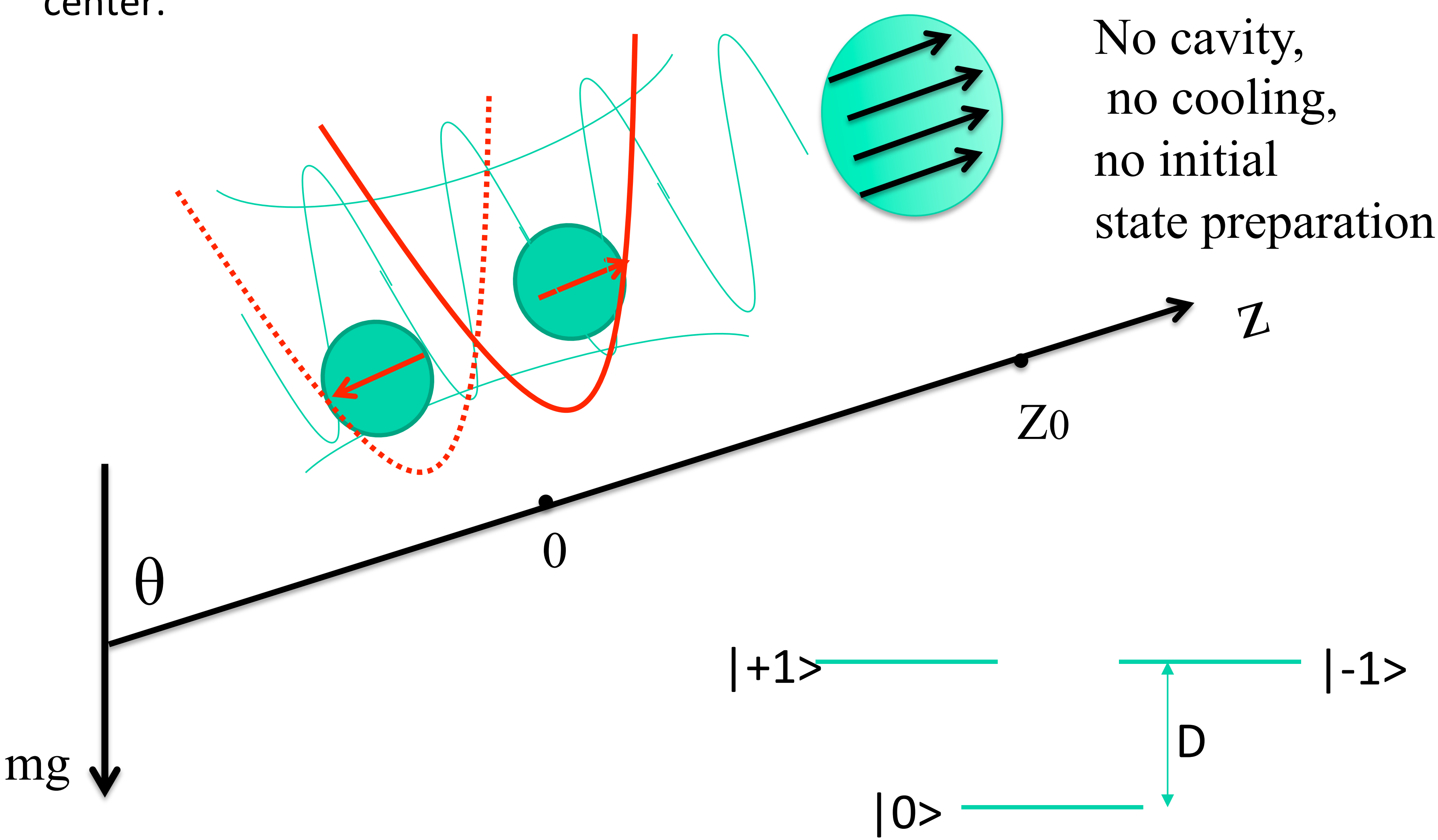
Armour, Blencowe, Schwab, PRL 2002.

Marshall, Simon, Penrose, Bouwmeester, PRL 2003.

Bose, PRL 2006.

Ramsey Interferometry with a Levitated Thermal Mesoscopic Object

Diamond bead trapped in an optical trap. The bead contains a spin-1 NV center.



Coupling between the spin and the motion

Calling (0,0,0) the position of the minimum of the potential, let us consider the magnetic field of a magnetized sphere with magnetic dipole **m**=(0,0,m_z) placed at the position (0, 0, z_0). Expanding it up to first order around the center of the trap, the magnetic field is:

$$B_x = -\frac{3\mu_0 m_z}{4\pi|z_0|^4} \cdot \frac{z_0}{|z_0|} x,$$

$$B_y = -\frac{3\mu_0 m_z}{4\pi|z_0|^4} \cdot \frac{z_0}{|z_0|} y,$$

$$B_z = \frac{\mu_0 m_z}{2\pi\,|z_0|^3} + \frac{3\mu_0 m_z}{2\pi|z_0|^4} \cdot \frac{z_0}{|z_0|} z.$$

The zeroeth order term in B gives a Zeeman splitting between |+1> and |-1>.

Coupling between the spin and the motion

The linear term in the expansion gives the following coupling between the spin and the vibrational motion:

$$H_{\rm int} = -\lambda[2\,S_z\,(c + c^\dagger) - A_x\,S_x\,(a + a^\dagger) - A_y\,S_y\,(b + b^\dagger)],$$

where:

$$A_x = \sqrt{\frac{\omega_z}{\omega_x}}, \qquad A_y = \sqrt{\frac{\omega_z}{\omega_y}},$$

and:

$$\lambda = \frac{3\mu_0 m_z z_0}{4\pi|z_0|^5}\,g_{NV}\,\mu_B\,\sqrt{\frac{\hbar}{2\,m\,\omega_z}}$$

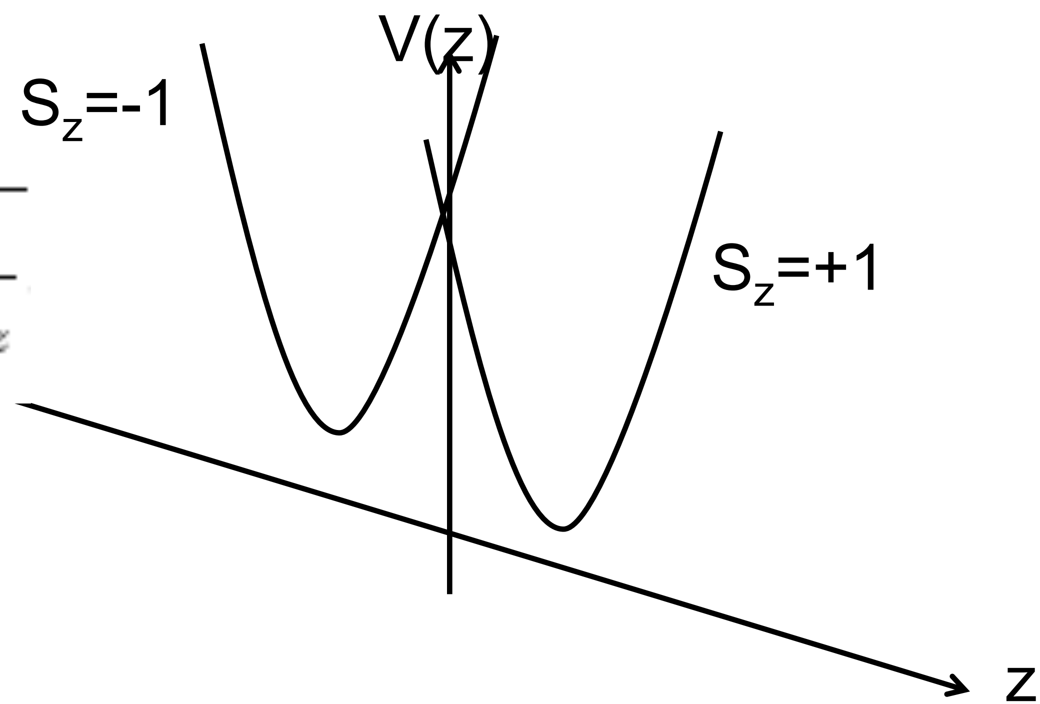
Spin Optomechanical coupling also derived in:
Z Yin, T Li, X Zhang, LM Duan, Physical Review A 88, 033614 (2013).
Also Rabl et. al. (2008), Oosterkamp et. al. (2008)

Spin-dependent displacement in a gravitational field

In the limit case of infinite ω_x and ω_y the Hamiltonian describes a conditional displacement of the trapping potential, whose direction depends on the value of S_z :

$$H = DS_Z^2 + \hbar\omega_Z c^+ c - 2\lambda S_Z (c + c^+) + 2\Delta\lambda (c + c^+)$$

$$\Delta\lambda = \frac{1}{2} mg \cos\theta \sqrt{\frac{\hbar}{2m\omega_z}}$$



The interferometric scheme

Suppose that the oscillator is initially in a coherent state |β> and that the spin is in the eigenstate |S_z=0>:

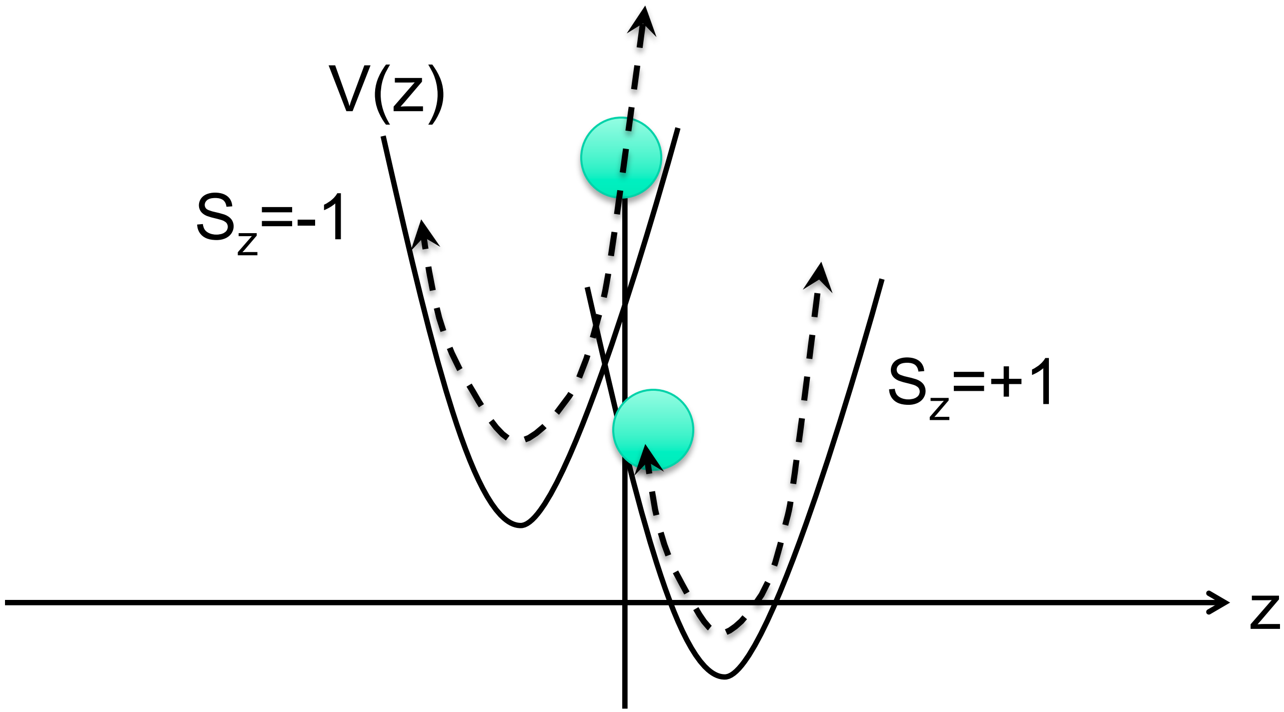
$$|\Psi(0)\rangle = |\beta\rangle\,|0\rangle$$

<u>Step 1</u>: apply a very rapid mw pulse which transforms the state of the spin according to:

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|+1\rangle + |-1\rangle\right)$$

So that we obtain the superposition of two states which oscillate in opposite directions.

Evolution of an *arbitrary* coherent state (at the time period *T* it comes back!)



$$(e^{-i\phi_+(t)}|\beta(t,+1)\rangle|+1\rangle + e^{-i\phi_-(t)}|\beta(t,-1)\rangle|-1\rangle)/\sqrt{2}$$

$$\Delta z(t) \;=\; \frac{8\lambda\delta_z}{\hbar\omega_z}(1\;-\;\cos\omega_z t)$$

Evolution

Step 2: the state evolves for time T equal to the period of the oscillator; the oscillation is different according to the spin, but at t=T the vibrational state is the same for both |+1> and |-1>

so that the spin state is separated from the vibrational state and is (up to a global phase):

$$|\Psi_{spin}\rangle = \frac{1}{\sqrt{2}}\left(|+1\rangle + e^{i\Delta\phi}\,|-1\rangle\right)$$

with: $\displaystyle\int_0^T \frac{mg\cos\theta\Delta z(t)dt}{\hbar} = \Delta\phi = -\frac{12\lambda\,\Delta\lambda}{\hbar^2\omega_z}T$

m = 10^(-17) Kg, omega_z = 100 kHz, Delta x=1 pm, Delta t ~ 1 mu s, Gradient ~ 10^4 T/m, we have **Delta phi ~ 1**

Measuring the phase shift due to gravitational potential difference

Step 3: apply the same very rapid mw pulse as in step 1, which, in the absence of the phase difference $\Delta\phi$ transforms the state of the spin according to:

$$\frac{1}{\sqrt{2}}\left(|{+1}\rangle + |{-1}\rangle\right) \rightarrow |0\rangle$$

The presence of $\Delta\phi$ gives a modulation of the population of $|S_z=0>$ according to:

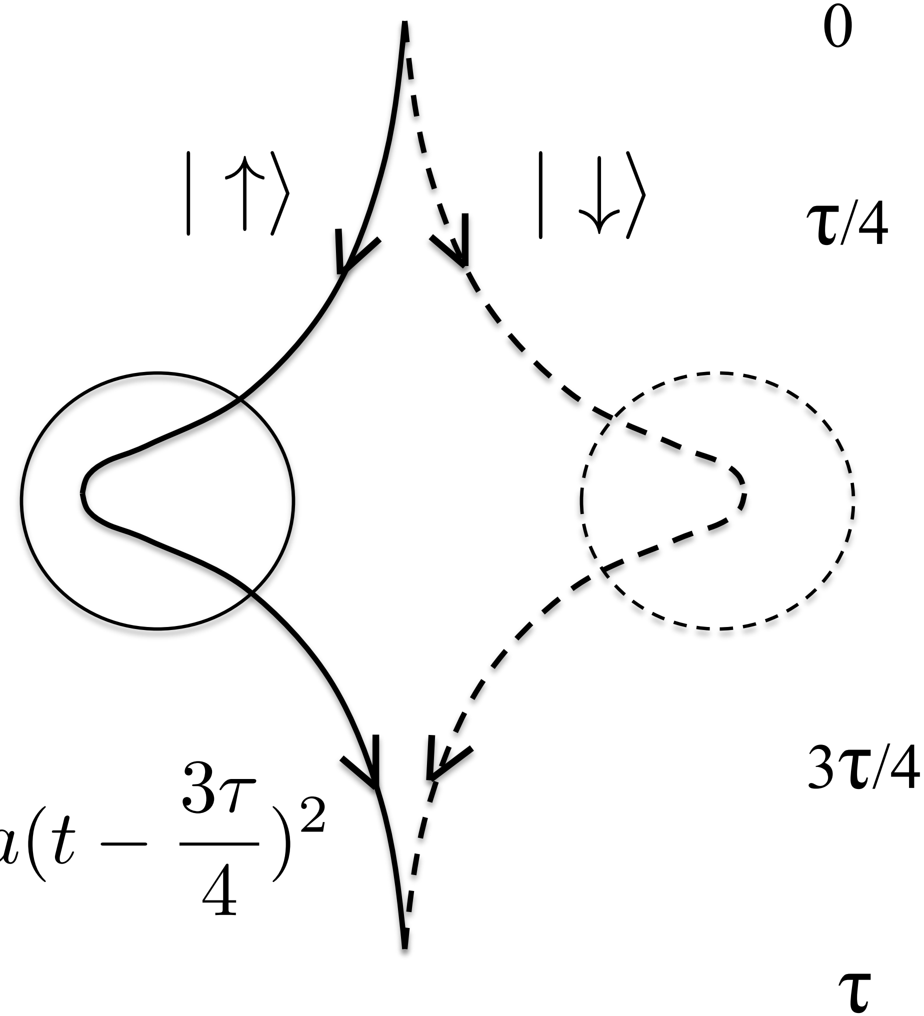
$$P_0(T + \Delta t_{pulse}) = \cos^2\frac{\Delta\phi}{2}$$

Free particle in an inhomogeneous magnetic field (acceleration $+a$ or $-a$)

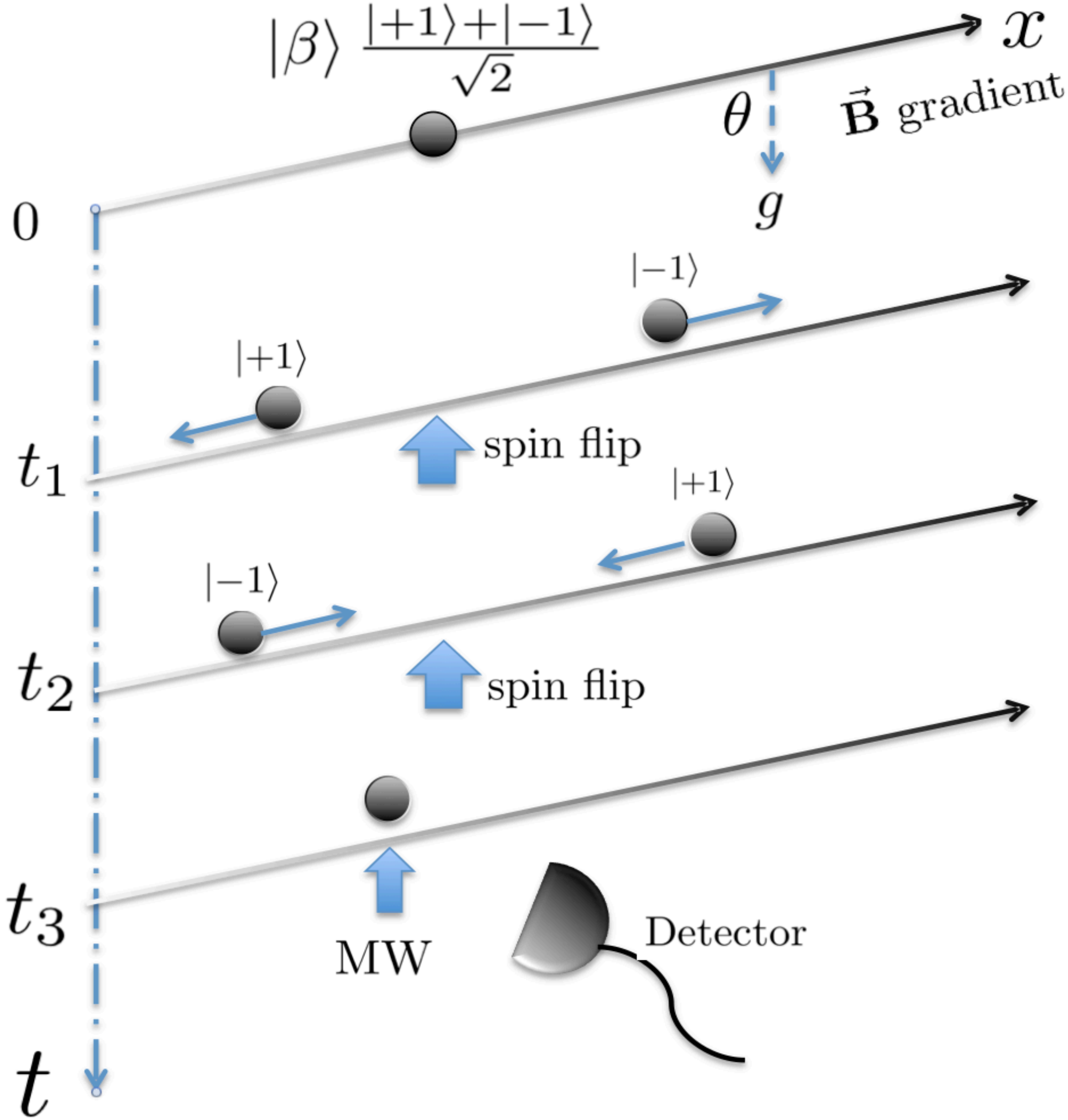
$$x_\sigma(t,j) = x_j(0) \pm \frac{1}{2}at^2$$

$$= \frac{a\tau}{4}(t - \frac{\tau}{4}) \mp \frac{1}{2}a(t - \frac{\tau}{4})^2$$

$$= \frac{1}{2}a(\frac{\tau}{4})^2 \mp \frac{a\tau}{4}(t - \frac{3\tau}{4}) \pm \frac{1}{2}a(t - \frac{3\tau}{4})^2$$



Free flight scheme able to achieve 100 nm separation among superposed components:



$$|\Psi(t_3)\rangle = \frac{1}{\sqrt{2}}|\psi(t_3)\rangle(|+1\rangle + e^{-i\phi_g}|-1\rangle)$$

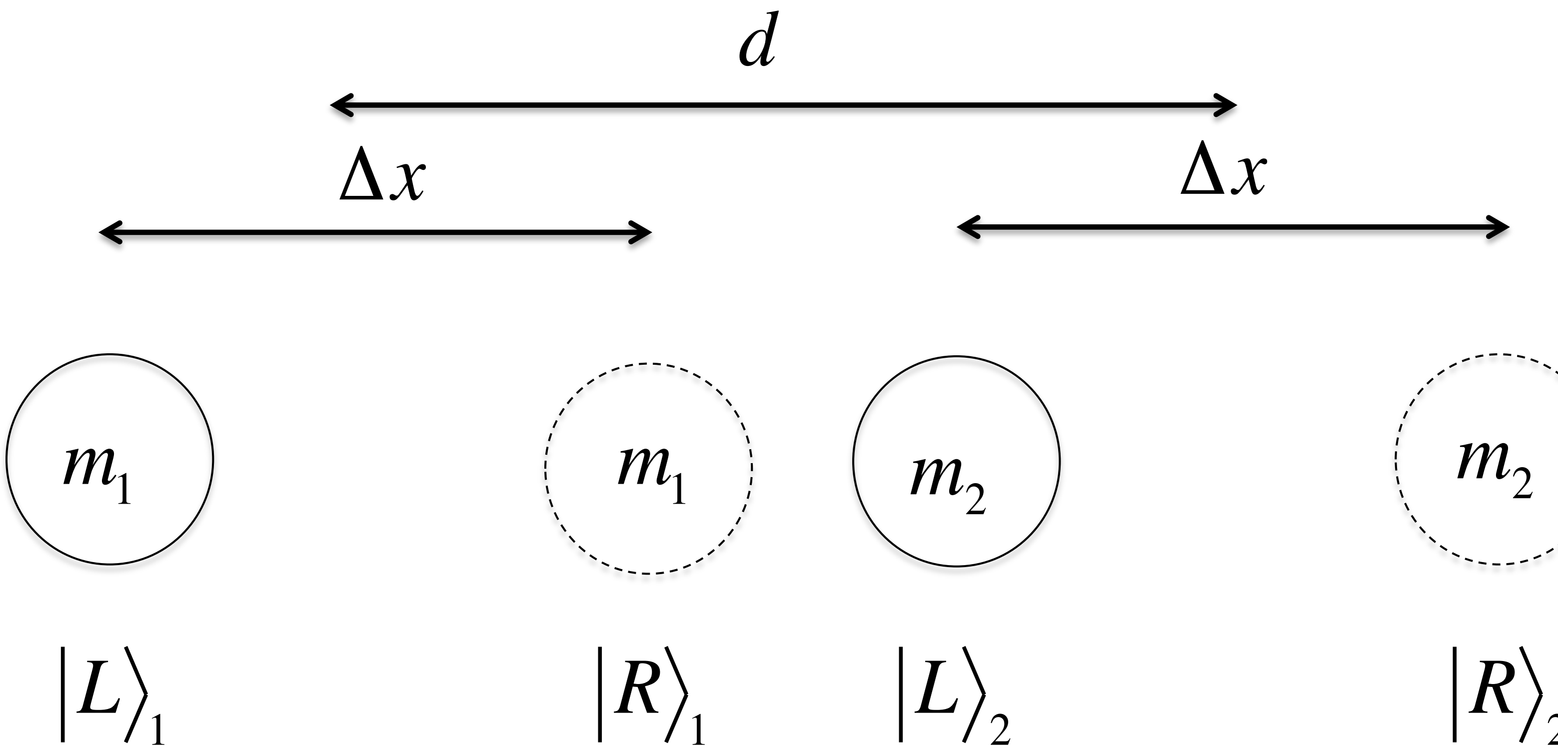
$$\langle x|\psi(t_3)\rangle = e^{-ip_0 x} e^{-[(x - x_0 - p_0 t_3/m - g\cos\theta t_3^2/2)^2/2(\sigma')^2]}$$

$$\phi_g = (1/16\hbar) g t_3^3 g_{\mathrm{NV}} \mu_B (\partial B/\partial x) \cos\theta$$

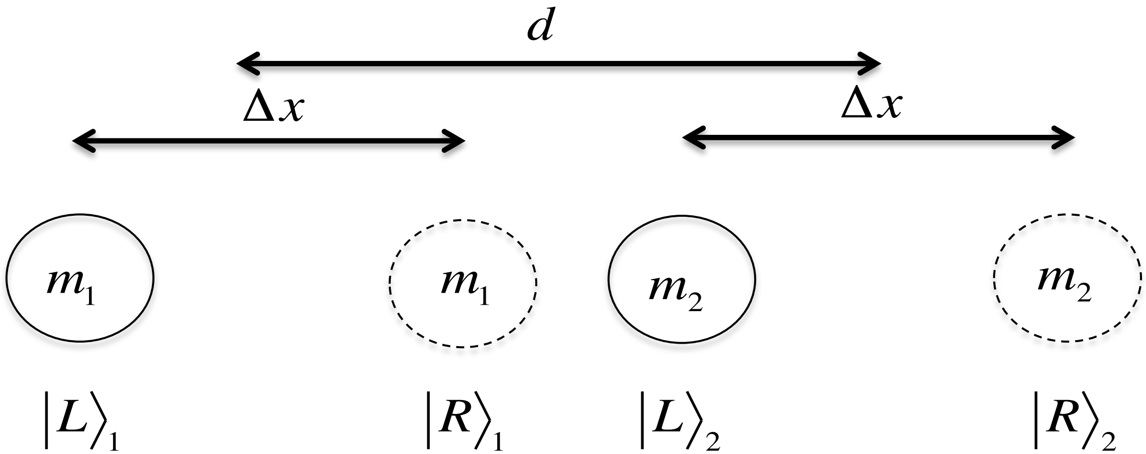
$$\Delta x_M = 2 \times \frac{1}{2m} g_{\mathrm{NV}} \mu_B \frac{\partial B}{\partial x} (t_3/4)^2$$

10^10 amu mass can be placed in a superposition of states separated by 100 nm.

A Schematic of two matter-wave interferometers near each other



Consider two neutral test masses *held* in a superposition, each exactly as a path encoded qubit (states |L> and |R>), near each other.



If they interact *only* through the gravitational force

$$|\Psi(t=0)\rangle_{12} = \frac{1}{\sqrt{2}}(|L\rangle_1 + |R\rangle_1)\frac{1}{\sqrt{2}}(|L\rangle_2 + |R\rangle_2)$$

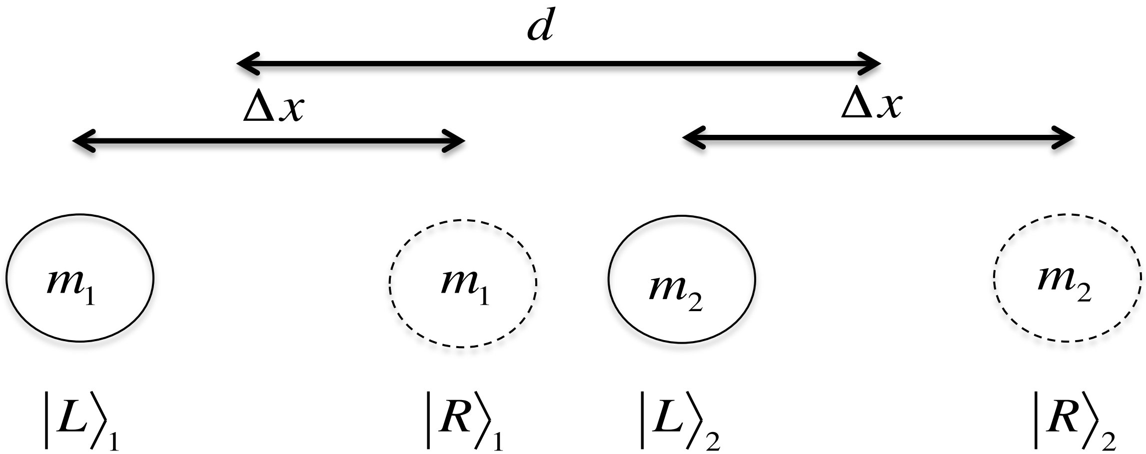
$$= \frac{1}{2}(|L\rangle_1|L\rangle_2 + |L\rangle_1|R\rangle_2 + |R\rangle_1|L\rangle_2 + |R\rangle_1|R\rangle_2)$$

$$\to |\Psi(t=\tau)\rangle_{12} = \frac{1}{2}(e^{i\phi_{LL}}|L\rangle_1|L\rangle_2 + e^{i\phi_{LR}}|L\rangle_1|R\rangle_2 + e^{i\phi_{RL}}|R\rangle_1|L\rangle_2 + e^{i\phi_{RR}}|R\rangle_1|R\rangle_2),$$

where

$$\phi_{RL} \sim \frac{Gm_1m_2\tau}{\hbar(d-\Delta x)}, \phi_{LR} \sim \frac{Gm_1m_2\tau}{\hbar(d+\Delta x)},$$

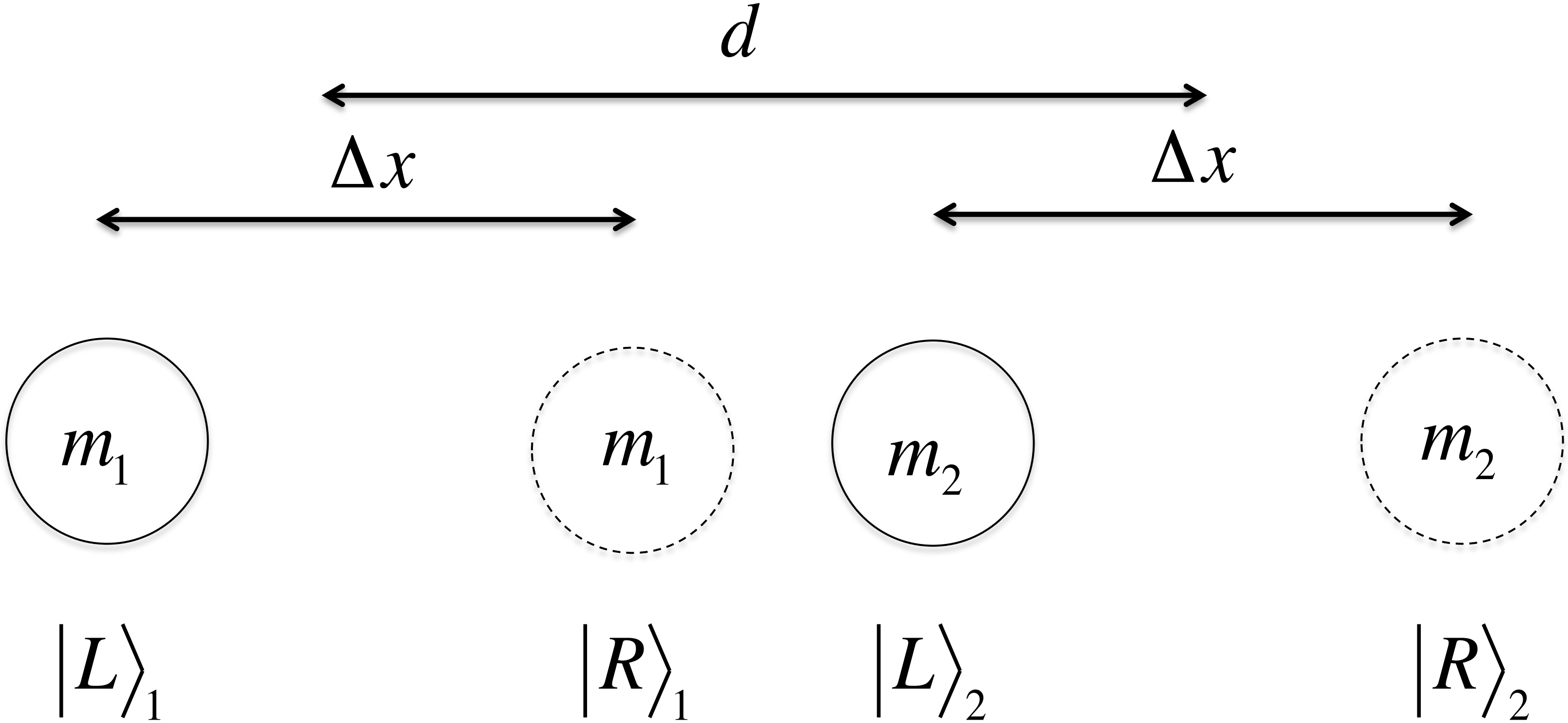
$$\phi_{LL} = \phi_{RR} \sim \frac{Gm_1m_2\tau}{\hbar d}$$



If they interact *only* through the gravitational force

$$
\begin{aligned}
|\Psi(t=\tau)\rangle_{12} &= \frac{1}{2}(e^{i\phi_{LL}}|L\rangle_1|L\rangle_2 + e^{i\phi_{LR}}|L\rangle_1|R\rangle_2 \\
&+ e^{i\phi_{RL}}|R\rangle_1|L\rangle_2 + e^{i\phi_{RR}}|R\rangle_1|R\rangle_2) \\
&= \frac{e^{i\phi_{RR}}}{\sqrt{2}}\{|L\rangle_1\frac{1}{\sqrt{2}}(|L\rangle_2 + e^{i\Delta\phi_{LR}}|R\rangle_2) \\
&+ |R\rangle_1\frac{1}{\sqrt{2}}(e^{i\Delta\phi_{RL}}|L\rangle_2 + |R\rangle_2)\}
\end{aligned}
$$

The above state is maximally entangled when $\Delta\phi_{LR} + \Delta\phi_{RL} \sim \pi$.



For

$$d - \Delta x << d, \Delta x,$$

we have

$$\Delta\phi_{RL} \sim \frac{Gm_1 m_2 \tau}{\hbar(d - \Delta x)} >> \Delta\phi_{LR}, \Delta\phi_{LL}, \Delta\phi_{RR}$$

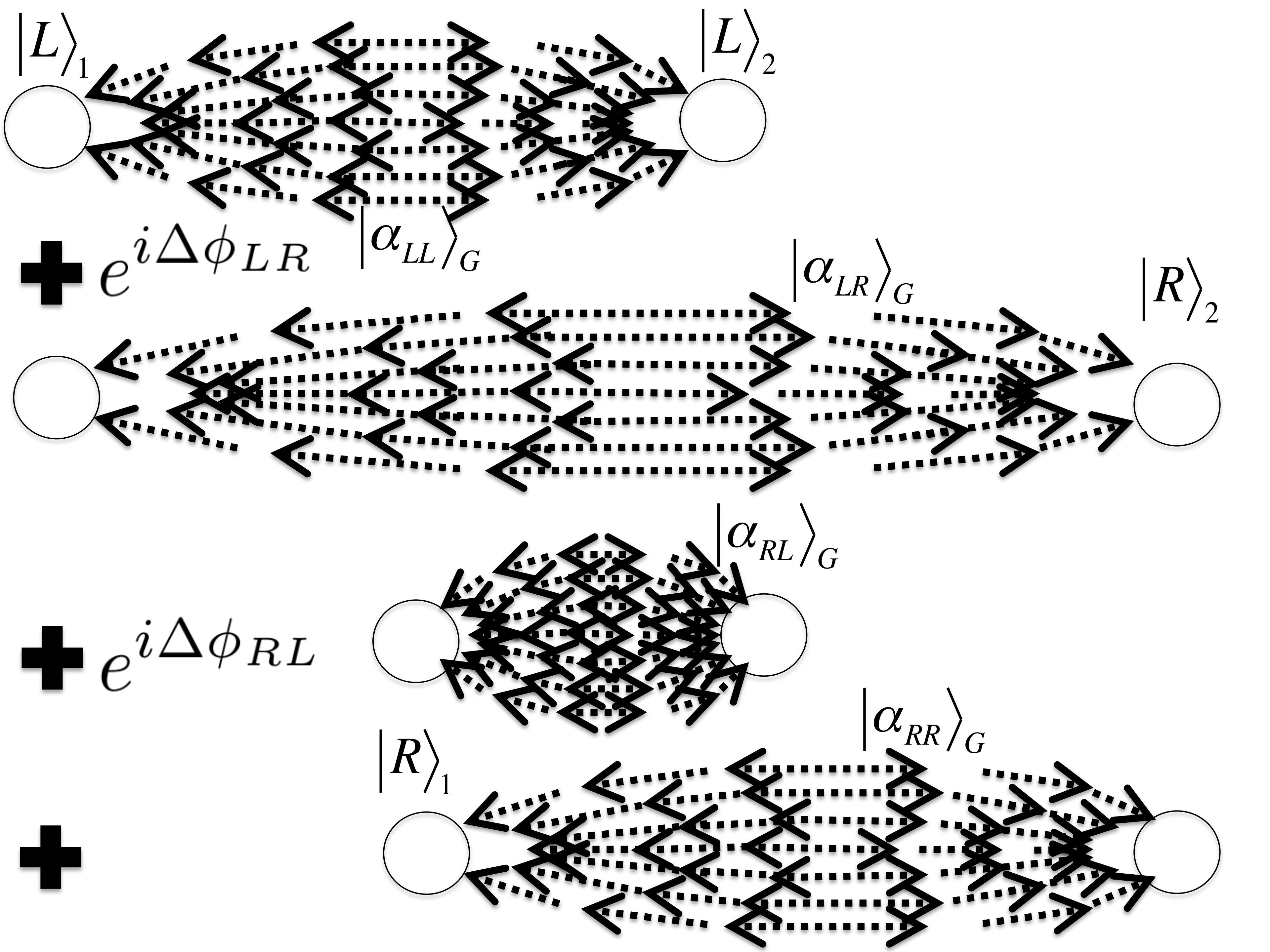
For

$$d - \Delta x << d, \Delta x,$$

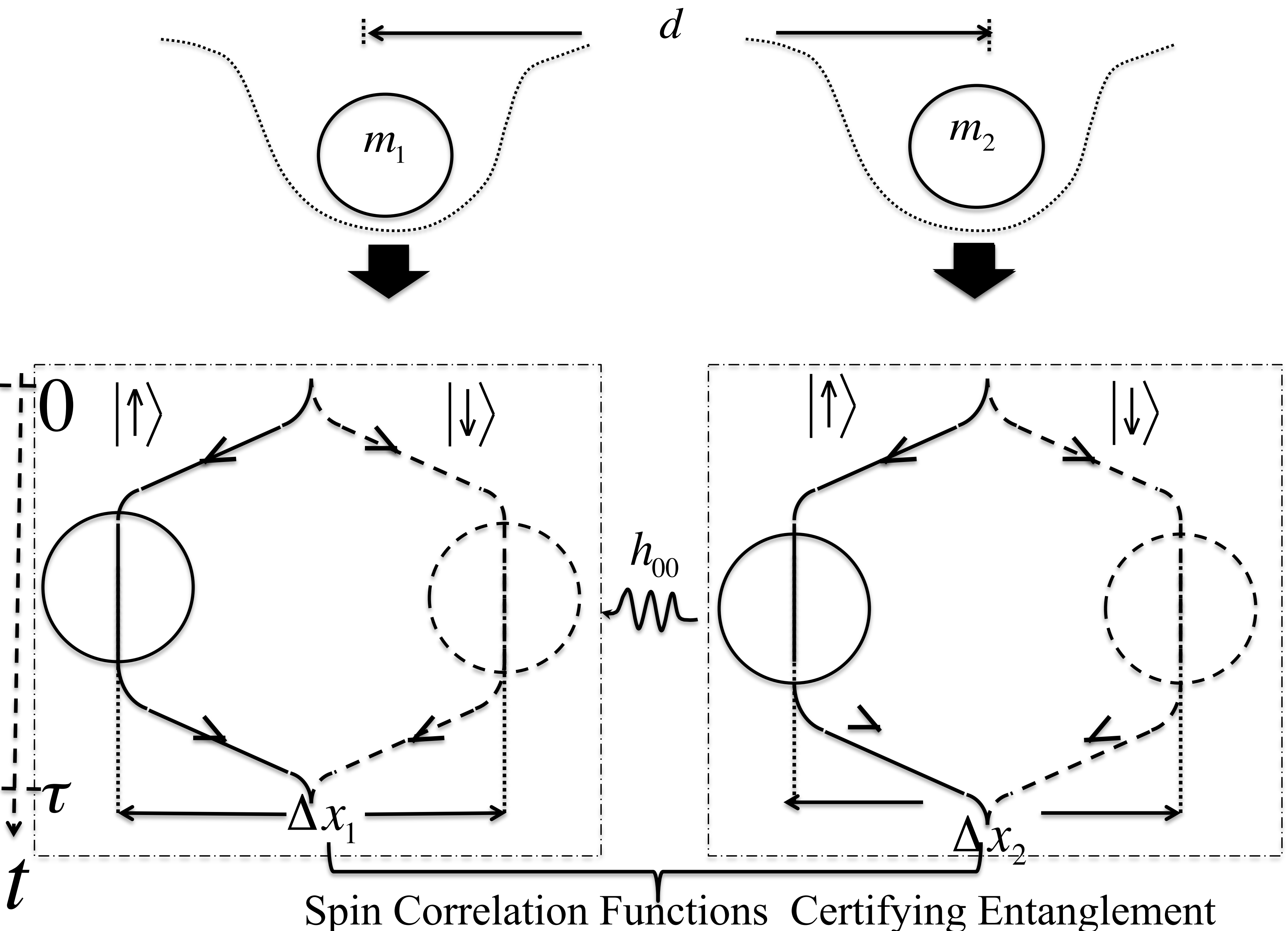
we have

$$\Delta\phi_{RL} \sim \frac{G m_1 m_2 \tau}{\hbar(d - \Delta x)} >> \Delta\phi_{LR}, \Delta\phi_{LL}, \Delta\phi_{RR}$$

For mass ~ 10^(-14) kg (microspheres), separation at closest approach of the masses ~ 100 microns (to prevent Casimir interaction), time ~ 1 seconds, Delta phi_{RL} ~ 1



$|L\rangle_1$
$|L\rangle_2$
$|\alpha_{LL}\rangle_G$
$+\ e^{i\Delta\phi_{LR}}$
$|\alpha_{LR}\rangle_G$
$|R\rangle_2$
$|\alpha_{RL}\rangle_G$
$+\ e^{i\Delta\phi_{RL}}$
$|\alpha_{RR}\rangle_G$
$|R\rangle_1$
$+$


d
m_1
m_2
0
$|\uparrow\rangle$
$|\downarrow\rangle$
$|\uparrow\rangle$
$|\downarrow\rangle$
h_{00}
τ
Δx_1
Δx_2
t
Spin Correlation Functions Certifying Entanglement

What does it imply in the context of **low energy effective field theory**?

$$\mathcal{H} = \sum_{j,\sigma} m_j c^2 a^\dagger_{\sigma,j} a_{\sigma,j} + \sum_{\mathbf{k}} \hbar\omega_k b^\dagger_{\mathbf{k}} b_{\mathbf{k}}$$
$$- \hbar \sum_{j,\mathbf{k},\sigma} g_{j,\mathbf{k}} a^\dagger_{\sigma,j} a_{\sigma,j} (b_{\mathbf{k}} e^{i\mathbf{k}.\mathbf{r}_{j,\sigma,t}} + b^\dagger_{\mathbf{k}} e^{-i\mathbf{k}.\mathbf{r}_{j,\sigma,t}})$$

Donohue, (for a long time.) Blencowe, PRL 2013

$$g_{j,\mathbf{k}} = m_j c^2 \sqrt{\frac{8\pi G}{\hbar c^3 k V}}$$

$$|\Psi_{\mathrm{mat+grav}}(t)\rangle = \frac{1}{2} \sum_{\sigma,\sigma'} a^\dagger_{1,\sigma} a^\dagger_{2,\sigma'} |0\rangle$$

$$\otimes \prod_{\mathbf{k}} e^{i \frac{|g_{1,\mathbf{k}} e^{i\mathbf{k}.\mathbf{r}_{1,\sigma,t}} + g_{2,\mathbf{k}} e^{i\mathbf{k}.\mathbf{r}_{2,\sigma',t}}|^2}{\omega_k} t} |\alpha_{\mathbf{k},\sigma,\sigma'}\rangle_{\mathbf{k}}$$

$$\alpha_{\mathbf{k},\sigma,\sigma'} = (\frac{g_{1,\mathbf{k}}}{\omega_k} e^{i\mathbf{k}.\mathbf{r}_{1,\sigma,t}} + \frac{g_{2,\mathbf{k}}}{\omega_k} e^{i\mathbf{k}.\mathbf{r}_{2,\sigma',t}})(e^{i\omega_k t} - 1)$$

Superpositions of *distinct (?)* coherent states of the gravitational field

Macro-realism Test:

$$C_{ij} \equiv \langle Q(t_i)Q(t_j)\rangle$$

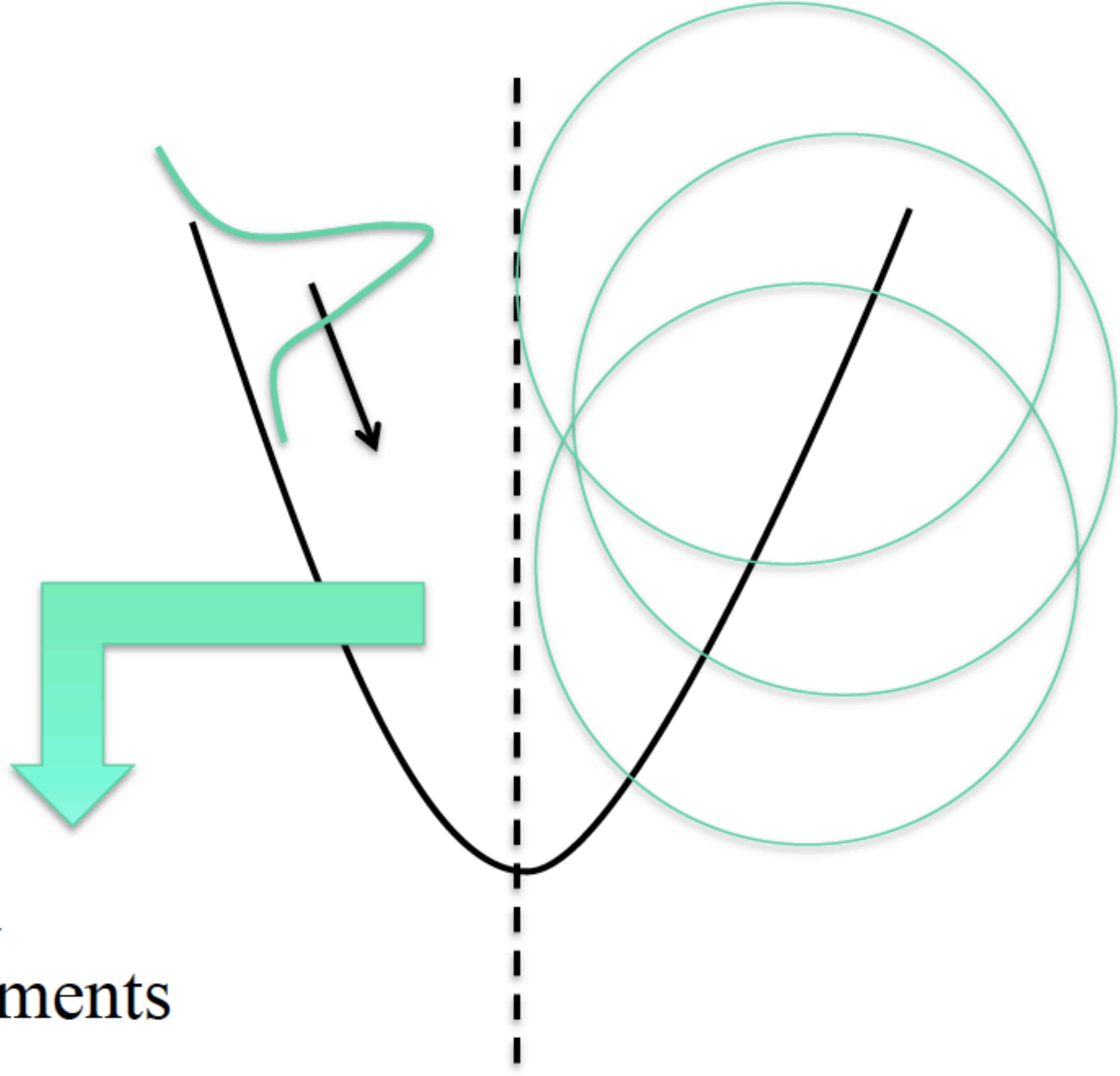
Macro-Realism + Non-Invasive = Leggett-Garg:

$$C \equiv C_{12} + C_{23} + C_{34} - C_{14} \leq 2$$

If Negative Result Measurment, then tests Macro-realism

LGI with NRM

Evolution of Coherent states in a HO



Retained Measurements

Linear Harmonic Oscillator

- Initial wavepacket is

$$\psi(x,0) = \sqrt{\frac{1}{\sqrt{2\pi}\sigma_0}} \exp\left(-\frac{(x-x_0)^2}{4\sigma_0^2} + \frac{ip_0(x-x_0)}{h}\right) \quad (3)$$

- Here one considers measuring localization of the particle. If the particle is found in the region between $x \to -\infty$ and $x = 0$, then the measurement outcome is denoted by $+1$. If the particle is found in the region between $x = 0$ and $x \to \infty$, then the outcome is denoted by -1.
- The above mentioned condition is satisfied by defining the following measurement operator

$$\hat{O} = \int_{-\infty}^{0} |x\rangle\langle x| dx - \int_{0}^{\infty} |x\rangle\langle x| dx \quad (4)$$

POST-MEASUREMENT STATE AT TIME t

- When the particle is found at the instant t in the region between $x \to -\infty$ and $x = 0$, the post-measurement state is given by

$$|\psi_{+}^{PM}(t)\rangle = \int_{-\infty}^{0} \psi(x', t)|x'\rangle dx' \tag{15}$$

- When the particle is found at the instant t in the region between $x = 0$ and $x \to \infty$, the post-measurement state is given by

$$|\psi_{-}^{PM}(t)\rangle = \int_{0}^{\infty} \psi(x', t)|x'\rangle dx' \tag{16}$$

FURTHER EVOLUTION OF THE STATE AFTER 1st MEASUREMENT

- If $+1$ result is obtained at, say, $t = t_1$, then the post-measurement state under the harmonic oscillator potential evolves into the following state at the instant $t = t_2$

$$|\psi_+^{PM}(t_2)\rangle = \int_{-\infty}^{\infty} K(x', t_1; x, t_2)\psi(x', t_1)_+^{PM}|x'\rangle dx' \quad (17)$$

- If -1 result is obtained at, say, $t = t_1$, then the post-measurement state under the harmonic oscillator potential evolves into the following state at the instant $t = t_2$

$$|\psi_-^{PM}(t_2)\rangle = \int_{-\infty}^{\infty} K(x', t_1; x, t_2)\psi(x', t_1)_-^{PM}|x'\rangle dx' \quad (18)$$

m(amu)	$\sigma_0(m)$	$p_0(kgm/s)$	$v_0(m/s)$	$A_{Cl}(m)$	C
10	3.9×10^{-8}	3.3×10^{-24}	2×10^2	10^{-4}	2.62
10^3	3.9×10^{-9}	3.3×10^{-23}	2×10	10^{-5}	2.58
10^6	1.2×10^{-10}	3.3×10^{-21}	2.0	10^{-6}	2.5
10^{10}	1.2×10^{-12}	3.3×10^{-21}	2×10^{-4}	10^{-10}	2.7
10^{20}	1.2×10^{-17}	3.3×10^{-15}	2×10^{-8}	10^{-14}	2.65

Conclusions

Long term motive: Enhancing the domain of investigation of the quantum superposition principle

M. Scala, M. S. Kim, G. W. Morley, P. F. Barker, S. Bose, Phys. Rev. Lett. **111**, 180403 (2013)
Free flight scheme: **C. Wan et. al.,** Phys. Rev. Lett. **117**, 143003 (2016).

Long term motive: To test the quantum nature of gravity.
S. Bose, A. Mazumdar, G. W.Morley, H. Ulbricht, M. Paternostro, P. F. Barker, A. Geraci, M. S. Kim, G. J. Milburn, arXiv (soon)

Long term motive: Testing the validity of macroscopic realism for increasingly more macroscopic objects.

S. Bose, D. Home and S. Mal, arXiv:1509.00196